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DEVELOPING A MATHEMATICAL MODEL FOR THE CUTTING FORCES PREDICTION DEPENDING ON THE CUTTING PARAMETERS

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Abstract

In this present study includes the experimentally determination of the cutting forces which occur during metal cutting, and developing a mathematical model for the cutting forces prediction depending on the cutting parameters variation. For this purpose, various cutting experiments were carried out under dry conditions using cutting tools have three different chip breaker forms. Machining tests were performed on AISI 1050 steel using three cutting speeds, two feed rates, and two depths of cuts. The cutting forces occurred during metal cutting were measured simultaneously by Kistler 9257B dynamometer. A practicable mathematical model for every one of the cutting force components was developed using the cutting force data measured by cutting tests. A good agreement between the results of the developed model for the cutting force components and the results of experimentally obtained the cutting force data was seen.

Key words: Cutting forces, cutting parameters, mathematical modelling

1. Introduction

The chip formation process is the result of plastic deformation has high strain rate and temperature effects. A number of different models for the cutting process have been developed in the literature, which can be classified into four major categories: analytical, experimental, mechanistic, and numerical methods. In order to define cutting process the various models were developed; some of them the cutting process successfully identified the exactly but exactly the right solutions could not be proven in some.

Oxley [1], Wang and Mathew [2], Arsecularatne et al. [3] and Moufki et al. [4] have used thermo-mechanical cutting model with a simplified assumptions containing the properties of materials such as effects of temperature, strain rate sensitivity and strain hardening to estimate the cutting forces for oblique cutting. A mechanistic modelling approach to predicting cutting forces for grooved tools in oblique cutting and turning has been developed by Parakkal et al. [5]. The model assumes the existence of an equivalent orthogonal cutting operation for any oblique operation. The effects of tool nose radius and chip flow in turning have been incorporated by defining the equivalent oblique cutting edge and a set of equivalent groove parameters. An analytical model of three-dimensional cutting developed by Strenkowski et al. [6] for predicting tool forces and the chip flow angle consists of coupling an orthogonal finite element cutting model based on an Eulerian approach with an analytical model of three-dimensional cutting developed by Usui et al. [7, 8] in which a minimum energy approach was used to determine the chip flow direction. With this approach, a predictive model of three-dimensional cutting can be developed that does not require measured data as input. Thomas and Beauchamp [9] investigated the collection and analysis of cutting-force, tool-vibration and tool-modal-parameter data generated by lathe dry turning of mild carbon steel samples at different speeds, feeds, depths of cut, tool nose radius, tool lengths and workpiece lengths. A shear zone model with parallel boundaries is used by Hayajneh et al. [10] to evaluate the dynamic cutting forces in orthogonal cutting. The cutting system was modelled using a single degree-of-freedom dynamic system where the variations of the cutting forces are represented by their total differentials. The influence of temperature on the flow shear stress of workpiece material is accounted for. Fang and Jawahir [11-13] concerned with the universal slip-

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line model and the corresponding hodograph for two-dimensional machining which can account for chip curl and chip back-flow when machining with a restricted contact tool. Machining parameters such as cutting force ratio, chip thickness, chip curl and chip back-flow was predicted based on a universal slip-line model [12], the derived maximum value principle to determine the stress condition in the plastic region [12], Dewhurst and Collins's matrix technique [14] for numerically solving slip-line problems and Powell algorithm [15] for the optimization of non-linear functions. Zhang et al. [16] developed an approximate mechanics model to predict the forces in cutting composites reinforced by unidirectional fibres, and also taken into account the geometric properties of cutting tool such as the rake angle and nose radius. Kurt's [17] work includes the experimentally determination of the cutting forces which occur during metal cutting, the analysis of the effects of cutting forces on the cutting tool by means of ANSYS software and the mathematically modelling of the primary cutting force and the stresses using the acquired findings. Korucu [18] examined the effects on cutting tool stresses of the chip breaker form in hole-drilling process. While the cutting forces were determined via experimental measurements, stresses on the cutting edge were analyzed by using the ANSYS software. Aksu [19] used the experimentally results with the different workpiece materials and cutting conditions to develop an analytical model in orthogonal and oblique cutting methods. Gündüz [20] predicted the cutting forces in turning with the method of fuzzy logic and artificial neural networks.

There are so many studies in the literature about the cutting force modelling in metal cutting. But the cutting force model that more efficient, more understandable concept, complex structures that do not contain analytical models can be obtained is considered. With the help of developed models; the events such as chip formation and flow, machinability, chip breaker design parameters and cutting tool strength, surface roughness and vibration can be examined in more detail is considered.

2. Material and Method

2.1. The Cutting Tests

In the cutting tests, AISI 1050 workpiece samples were used as workpiece material. The coated cemented carbide inserts have PM, PR, and QM chip breaker forms with an ISO designation SNMG 120408 (Sandvik, 4225 grade) were used in the cutting tests. The inserts were mounted on a tool holder with an ISO designation PSBNR 2525M12 (Sandvik). Before starting experiments, the outer surface of the test specimens was machined in order to minimize the roll forming effects such as the outer surface hardness.

Without a coolant, cutting parameters as three different cutting speeds and two different feed rates and depths of cut were used during the measurement of cutting forces (Table 1). Cutting tests were carried out on a JOHNFORD T35 CNC lathe. The test specimens were machined 30 mm longitudinal. The principal cutting force, the feed force and the passive/radial force components (F_C, F_f, and F_p, respectively) according to the change in cutting parameters were measured simultaneously by Kistler piezoelectric dynamometer Type 9257B. After cutting experiments, the average values for F_C, F_f, and F_p force components were determined basis of the stable cutting region's start and end values.

Table 1. The cutting parameters used in the metal cutting tests

Workpiece material	: AISI 1050
Tool holder	: PSBNR 2525M12
Insert	: SNMG 120408
Chip breaker forms (rake angles)	: PM (7º), PR (8º), QM (9º)
Cutting parameters	
Cutting speed (V, m/min)	: 300-350-400
Feed rate (f, mm/rev)	: 0,2-0,4
Depth of cut (a, mm)	: 1-2

2.2. Modelling procedure

The validity of the model will be developed for F_C, F_f, and F_p components in terms of reliability, the cutting parameters directly affect the cutting force are important [17, 18]. Therefore, cutting parameters such as cutting speed, feed rates, depth of cut and rake angle of chip breaker form which affect directly cutting force were taken into consideration in the developed force model. Hence, with this study by using the basic cutting parameters; the experiment in terms of economic costs, complex analytic structure does not contain, understandable, high-reliability, real data can be obtained, the actual cutting conditions can be applied easily to the development of cutting force model is intended. Despite Kurt's model [17] based cutting speed (*V*), feed rate (*f*), depth of cut (*a*) is a general knowledge-based force model for cutting forces, Gürbüz [21] also revealed the effects of the chip breaker geometry on the cutting forces. Therefore, considering the effects of chip breaker geometry, it is focused on a mathematical model containing *V*, *f*, *a*, and the rake angle of the chip breaker (γ). The developed model for cutting force components is shown Eq.1:

$$F_i = c \cdot V^{q_1} \cdot f^{q_2} \cdot a^{q_3} \cdot \gamma^{q_4} \qquad (1)$$

where F_i, the cutting force components (F_C, F_f, and F_p); *c*, the cutting force component (or model) constant; q_1, q_2, q_3, q_4 exponents for cutting speed (*V*, m/min), feed rate (*f*, mm/rev), depth of cut (*a*, mm) and the rake angle of the chip breaker (γ, degree), respectively. All *c* and *q* values shown in Eq.1 should be determined separately for each cutting force component (F_C, F_f, and F_p). The cutting force component constant, c and q_1, q_2, q_3, q_4 exponents have to be determining for the solution of Eq. 1. For this goal, Eq. 1 was transformed to logarithm form as follows

$$log\ F_i = log\ c + q_1 \cdot log\ V + q_2 \cdot log\ f + q_3 \cdot log\ a + q_4 \cdot log\ \gamma \qquad (2)$$

and then Eq. 2 was transformed to linear form as follows

$$Y = C_0 \cdot X_0 + C_1 \cdot X_1 + C_2 \cdot X_2 + C_3 \cdot X_3 + C_4 \cdot X_4 + \varepsilon_i \qquad (3)$$

where *Y*, the logarithm value of the cutting force component (e.g. $log\ F_C$); C_i (i = 1,2,3,4) to be calculated equation constants (q_1, q_2, q_3, q_4 exponents, respectively); X_1, X_2, X_3, X_4, logarithmic values of *V*, *f*, *a*, γ, respectively; ε_i, (i=1,2,3) the error term. C_0 and X_0 were used to calculate *c* constant, and X_0 was assumed as an imaginary factor and its value was selected 1. The matrix form of Eq. 3 is

$$Y = X \cdot C + e \qquad (4)$$

where

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdots \\ Y_n \end{bmatrix},\ C = \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_n \end{bmatrix},\ e = \begin{bmatrix} e_1 \\ e_2 \\ \cdots \\ e_n \end{bmatrix} \text{ and } X = \begin{bmatrix} 1 & X_{21} & .. & X_{K1} \\ 1 & X_{22} & .. & X_{K2} \\ .. & .. & .. & .. \\ 1 & X_{2n} & .. & X_{Kn} \end{bmatrix}$$

where *n* and *K*, the number of test and the parameter, respectively. *Y*, *X*, *C* and *e* show the dependent variables vector with $n \times 1$ dimension, the independent variables matrix with $n \times K$ dimension, the model constant vector with $K \times 1$ dimension and the error term vector with $n \times 1$ dimension, respectively.

A mathematical model called as regression model that the cause-effect relationship between two or more variables related to that subject in order to make predictions or estimates, and characterized by a statistical analysis technique carried out using regression analysis was used for developed cutting force model (Eq. 1). For the model is especially take place multivariate linear regression analysis of the particular type of regression analysis, the least squares method was applied to the model constants used in the identification of Eq. 1. In the least squares technique, the difference of the real values obtained test results with the modelled theoretical values is minimized. For the solution of Eq. 4 (indirectly Eq. 1), following format is used:

$$C = [X' \cdot X]^{-1} \cdot X' \cdot Y \qquad (5)$$

In Eq. 5; *C*, will be find the coefficient vector; *X*, independent variable matrix; and *Y*, the test results vector for the each cutting force component.

3. Result and Discussion

The cutting force component (or model) constant *c*, and exponents q_1, q_2, q_3, q_4 (for cutting speed, feed rate, depth of cut, and the rake angle of the chip breaker, respectively) used in the developed cutting force model (Eq. 1) were calculated by the solution of Eq. 5 according to the cutting experiments carried out. Accordingly, the model constant *c* and q_i (i = 1,2,3,4) exponents used in the developed cutting force model based *V*, *f*, *a*, and γ for the principal cutting force, the feed force and the passive/radial force components (F_C, F_f, and F_p, respectively) were calculated separately for F_C, F_f, and F_p, respectively. The model constants (*c* and q_i) calculated based the developed cutting force model for F_C, F_f, F_p, the mean errors that the average of differences between the measured cutting force results and the calculated cutting force component results, and the coefficient of determination, R^2, used to control of the developed model suitability to the observation values, are given Table 2.

Table 2. The model constants, the mean errors, and R^2 values

F_i	*c*	q_1	q_2	q_3	q_4	Error (± %)	R^2
F_C	1541,10783	-0,01424	0,82494	0,92336	0,09338	2,619	0,9927
F_f	366,98412	-0,04435	0,44725	1,09105	0,20548	6,353	0,9380
F_p	658,19753	-0,04042	0,64384	0,54516	0,06059	9,296	0,8936

As a result of the calculations based the parameters (q_1, q_2, q_3, q_4 exponents and *c* constants) in the model developed for each of the cutting forces F_C, F_f, F_p, it is remarkable that the mean errors for the all chip breaker forms remains within the boundaries of error ±10 % (±10 % of the measured cutting force) (Table 2).

The developed model for cutting force prediction model specifically can be said that the errors for the principal cutting force results are less than ±3 %. It is shown that the errors for the feed force (F_f) and the passive/radial force (F_p) are approximately ±6,4 % and ±9,3 %, respectively. In contrast, the coefficients of determination, R^2, for the cutting forces F_C, F_f, F_p are approximately 0,99; 0,94 and 0,90, respectively (Table 2).

The distributions of the calculated cutting force results by the model with the correlations of the experimentally measured results for the chip breaker forms PM, PR, and QM with the rake angle of 7º, 8º, 9º, respectively, are shown in Fig. 1 to Fig. 3, respectively.

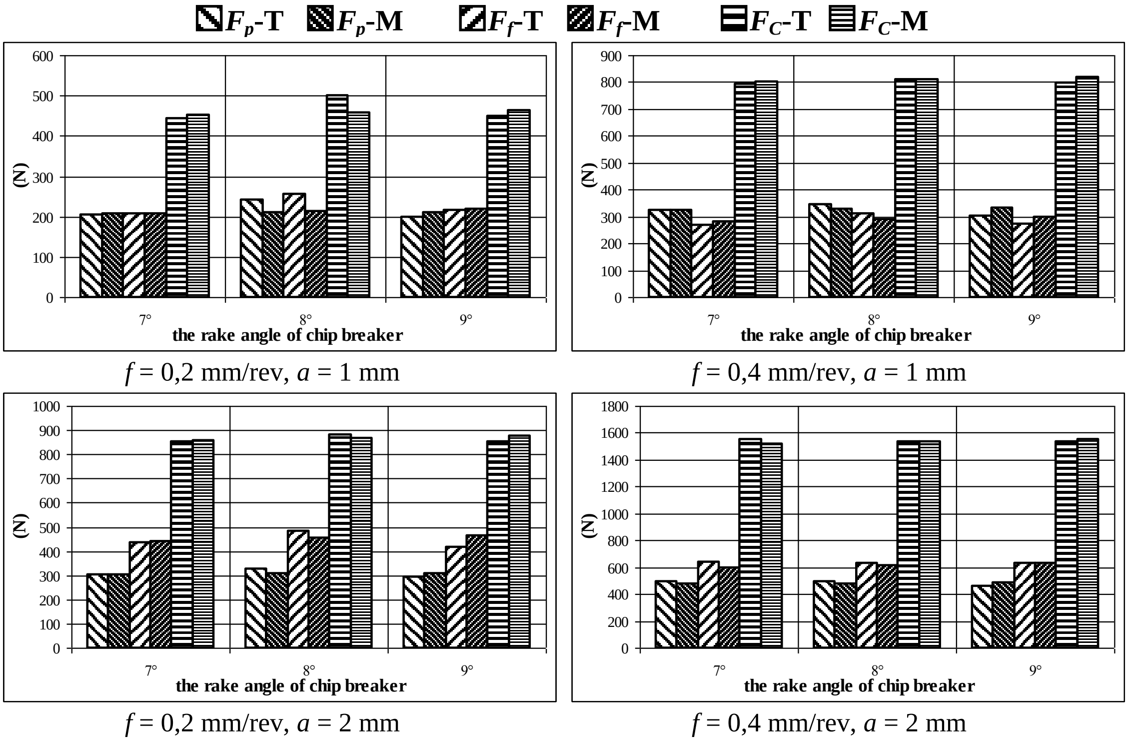

Figure 1. The model distributions for V = 300 m/min

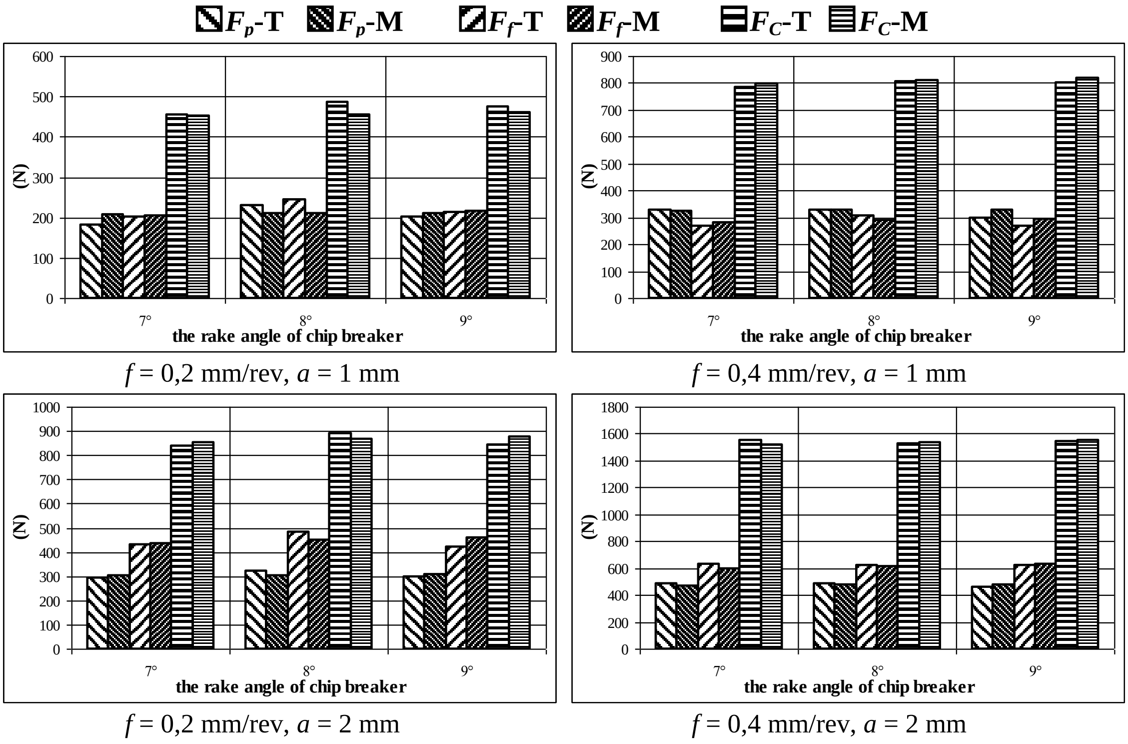

Figure 2. The model distributions for V = 350 m/min

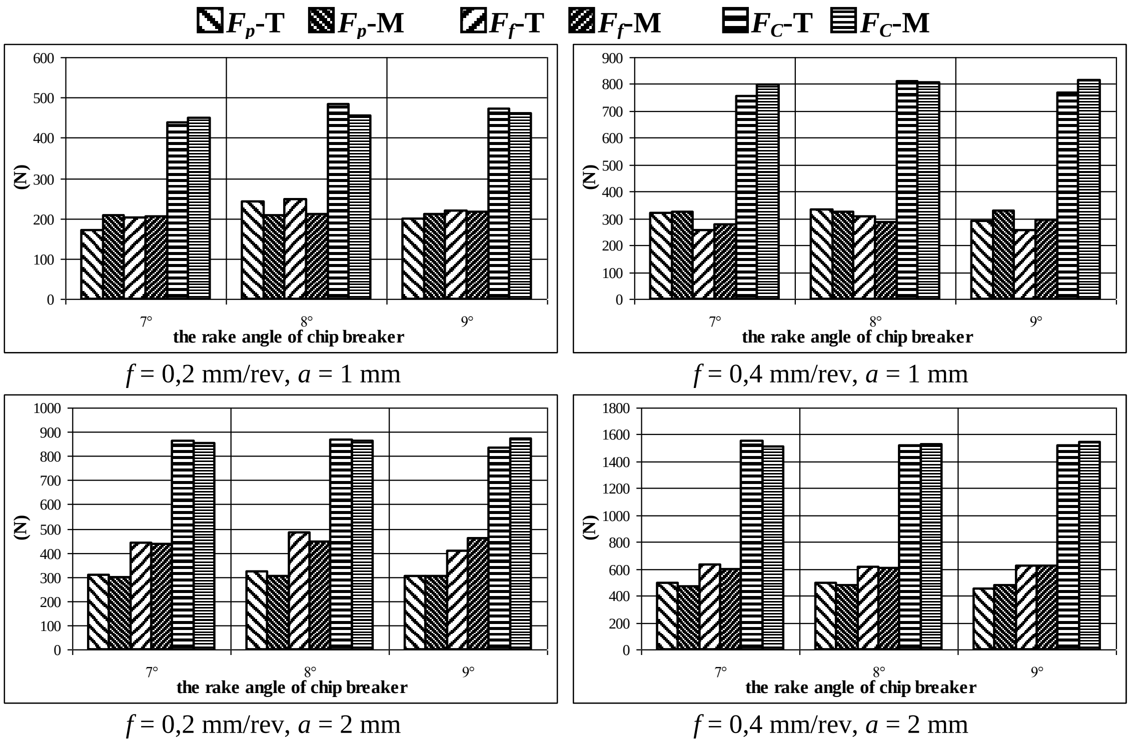

Figure 3. The model distributions for V = 400 m/min

In generally, while the measured cutting forces decrease with an increase in the cutting speed, it is observed the opposite situation with the increase in the feed rate and the depth of cut with the opposite situation is observed. It can be said that the cutting forces measured with the chip breaker form PR have the rake angle of 8º for all cutting speed is the highest.

For the passive force, while the error level that between the experimentally measured cutting force results (F_p-T) and the calculated cutting force results (F_p-M) with the chip breaker forms PM (7º) and PR (8º) for all cutting speeds under the light cutting conditions (feed rate 0,2 mm/rev and depth of cut 1 mm) are approximately ±10-15 %, the error level under the other cutting conditions is approximately ±3-4 %.

The error level that between the test results (F_f-T) and the calculated cutting force results (F_f-M) for the chip breaker form PR (8º) under the light cutting conditions for all cutting speeds, and for the chip breaker form QM (9º) under the conditions that V=300 m/min, f= 0,2 mm/rev, a=2 mm and V=400 m/min, f= 0,4 mm/rev, a=1 mm are approximately ±10-15 %, respectively.

In terms of the principal cutting force F_C, a good agreement between the results of the developed model and the results of experimentally obtained the cutting force data was seen (the error level is approximately ±2-3 %).

Outside the specified cutting conditions; the error level between the results of the developed model and the results of experimentally obtained for each of the cutting forces F_C, F_f, F_p for all the chip breaker forms under the terms of all cutting is approximately ±3-4 %.

4. Conclusion

By means of the carried out metal cutting tests on AISI 1050 workpiece with coated cemented carbide inserts have PM, PR, and QM chip breaker forms with an ISO designation SNMG 120408 (Sandvik, 4225 grade) and a tool holder PSBNR 2525M12 (Sandvik), a mathematical model based cutting parameters such as cutting speed, feed rate, depth of cut and the rake angle of the chip breaker for cutting force components was developed due to the experimental work costs are very high. The model constants used in the developed cutting force model were calculated separately using the least squares method, in the context of multivariate linear regression analysis, for each of the experimentally measured F_C, F_f, and F_p. It has been shown that the relevant cutting force component values can be estimate with the developed model based on the cutting parameters used in metal cutting tests. When the model results were compared with the results of experimentally measured results, it was found that model results had good agreement with the experimentally results. The developed force model specifically in a way more detailed review of events such as machinability, chip breaker design parameters, and surface roughness is thought to be beneficial.

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